MATHS – PAPER – JEE MAINS – 2019

9TH JANUARY – 2019 – SHIFT 1

1. There are 5 girls and 7 boys. A team of 3 boys and 2 girls is to be formed such that no two specific boys are in the same team. Number of way to do so

   (1) 400 (2) 250 (3) 200 (4) 300

Answer: (4)

Solution:

Total number of teams $= {}^{7}C_{3}\times{}^{5}C_{2} = 35\times10 = 350$

Let A, B be the specific boys.

Number of teams with these two boys in the same team $= {}^{5}C_{1}\times{}^{5}C_{2} = 5\times10 = 50$

$\therefore$ Required number of ways $= 350 - 50 = 300$

2. The equation $x^2+2x+2=0$ has roots α and β. Then value of $\alpha^{15}+\beta^{15}$ is

   (1) 512 (2) 256 (3) -512 (4) -256

Answer: (4)

Solution:

$$(x+1)^2 = -1 \Rightarrow x+1 = \pm i$$

$$x = -1+i,\ -1-i$$

$$\alpha = -1+i,\ \beta = -1-i$$

$$\alpha = \sqrt{2}e^{i\left(\frac{3\pi}{4}\right)},\ \beta = \sqrt{2}\,e^{i\left(\frac{-3\pi}{4}\right)}$$

$$\alpha^{15}+\beta^{15} = \left(\sqrt{2}\right)^{15}\left(e^{i\frac{45\pi}{4}} + e^{i\left(\frac{-45\pi}{4}\right)}\right)$$

$$= 2^{\frac{15}{2}}\left(2\cos\left(\frac{45\pi}{4}\right)\right)$$

$$= 2^{\frac{15}{2}}\left(2\left(\frac{-1}{\sqrt{2}}\right)\right)$$

$$= -2^8 = -256$$

3. $\int_0^{\pi} |\cos x|^3 dx$ is equal to

(1) $\frac{4}{3}$ (2) $\frac{2}{3}$ (3) 0 (4) $\frac{-8}{3}$

Answer: (1)

Solution:

$$\int_0^{\pi} |\cos x|^3 dx = 2\int_0^{\frac{\pi}{2}} \cos^3 x.dx$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{3\cos x + \cos 3x}{4}$$

$$= \frac{1}{2}\left(3\sin x + \frac{\sin 3x}{3}\right)_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left(3 - \frac{1}{3}\right) = 2\times\frac{8}{3} = \frac{4}{3}$$

4. If $x^2 \neq n\pi + 1.\ n \in N$ then $\int x\sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}}dx$ is equal to

(1) $\ln\cos\left(\frac{x^2-1}{2}\right) + c$ (2) $\frac{1}{2}\ln\cos\left(\frac{x^2-1}{2}\right) + c$

(3) $\ln\sec\left(\frac{x^2-1}{2}\right) + c$ (4) $\frac{1}{2}\ln\sec\left(\frac{x^2-1}{2}\right) + c$

Answer: (3)

Solution:

Let $x^2 - 1 = t \quad \Rightarrow x\,dx = \frac{dt}{2}$

$$\int \sqrt{\frac{2\sin t - \sin 2t}{2\sin t + \sin 2t}} \cdot \frac{dt}{2} \quad = \frac{1}{2}\int \sqrt{\frac{2\sin t(1-\cos t)}{2\sin t(1+\cos t)}}.dt$$

$$= \frac{1}{2}\int \tan\frac{t}{2}.dt$$

$$= \log\left|\sec\frac{t}{2}\right| + c$$

$$= \log\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$$

5. If $\vec{a} = i - j, \vec{b} = i + j + k$ are two vectors and $\vec{c}$ is another vector such that $\vec{a}\times\vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$ then $\left|\vec{c}\right|^2 =$

(1) 9 (2) 8 (3) $\frac{19}{2}$ (4) $\frac{17}{2}$

Answer: (3)

Solution:

$\vec{a}\times\vec{c} + \vec{b} = 0$

$\vec{a}\times\vec{c} = -\vec{b}$

$\vec{a}\times\left(\vec{a}\times\vec{c}\right) = \vec{b}\times\vec{a}$

$\left(\vec{a}.\vec{c}\right)\vec{a} - \left(\vec{a}.\vec{a}\right)\vec{c} = \vec{b}\times\vec{a}$ … (1)

$\vec{a}.\vec{c} = 4;\ \vec{a}.\vec{a} = 2$ … (2)

$$\vec{b}\times\vec{a}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & -1 & 0\end{vmatrix}=\hat{i}+\hat{j}-2\hat{k} \qquad \ldots (3)$$

From (1), (2), (3)

$$4\vec{a}-2\vec{c}=\hat{i}+\hat{j}-2\hat{k}$$

$$-2\vec{c}=\hat{i}+\hat{j}-2\hat{k}-4\left(\hat{i}-\hat{j}\right)$$

$$-2\vec{c}=-3\hat{i}+5\hat{j}-2\hat{k}$$

$$4\left|\vec{c}\right|^2=38\Rightarrow\left|\vec{c}\right|^2=\frac{19}{2}$$

6. $f(x)=\begin{cases}5 & ; \quad x\le 1\\ a+bx & ; \quad 1<x<3\\ b+5x & ; \quad 3\le x<5\\ 30 & ; \quad x\ge 5\end{cases}$ then

(1) $f(x)$ is discontinuous $\forall a\in R,\ b\in R$ (2) $f(x)$ is continuous if $a=0\ \&\ b=5$

(3) $f(x)$ is continuous if $a=5\ \&\ b=0$ (4) $f(x)$ is continuous if $a=-5\ \&\ b=10$

Answer: (1)

Solution:

$$f\left(1^-\right)=f\left(1^+\right)$$

$$a+b=5 \qquad \ldots (1)$$

$$f\left(3^+\right)=f\left(3^-\right)$$

$$b+15-=a+3b \qquad \ldots (2)$$

$$f\left(5^+\right)=f\left(5^-\right)$$

$b+25=30$ … (3)

From (3), b = 5

From (1), $a=0$ & from (2), $a=5$

$\therefore f(x)$ is discontinuous $\forall\, a\in R,\, b\in R$

7. Average height & variance of 5 students in a class is 150 and 18 respectively. A new student whose height is 156 cm is added to the group. Find new variance.

(1) 20 (2) 22 (3) 16 (4) 14

Answer: (4)

Solution:

Let 5 students are x_1, x_2, x_3, x_4, x_5

Given $\bar{x}=\frac{\sum x_i}{5}=150 \Rightarrow \sum_{i=1}^{5} x_i = 750$ … (1)

$\frac{\sum x_i^2}{5}-\left(\bar{x}\right)^2=18 \Rightarrow \frac{\sum x_i^2}{5}-(150)^2=18$

$\Rightarrow \sum x_i^2=(22500+18)\times 5 \Rightarrow \sum_{i=1}^{5} x_i^2 = 112590$ … (2)

Height of new student = 156 (Let X_6)

Now $x_1+x_2+x_3+x_4+x_5+x_6=750+156$

$\bar{x}_{new}=\frac{x_1+x_2+x_3+x_4+x_5+x_6}{6}=\frac{906}{6}=151$ … (3)

Variance (new) $=\frac{\sum x_i^2}{6}-\left(\bar{X}\right)^2$

$=\frac{x_1^2+x_2^2+x_3^2+x_4^2+x_5^2+x_6^2}{6}-(151)^2$

From equation (2) and (3)

Var (new) $= \dfrac{112590+(156)^2}{6}-(151)^2 = 22821-22801 = 20$

8. a, b, c are in G.P, $a+b+c=bx$, then x can not be

(1) 2 (2) -2 (3) 3 (4) 4

Answer: (1)

Solution:

Let the terms of G.P be $\frac{a}{r}, a, ar$

$\therefore \frac{a}{r}+a+ar=ax$

$\Rightarrow x=r+\frac{1}{r}+1$

But $r+\frac{1}{r}\geq 2$ or $r+\frac{1}{r}\leq -2$ (using A.M,G.M inequality)

$\therefore x-1\geq 2 \ or \ x-1\leq -2$

$\Rightarrow x\geq 3 \ or \ x\leq -1$

9. $\left\{\frac{2^{403}}{15}\right\}=\frac{k}{15}$ then find k. (where {.} denotes fractional part function)

(1) 2 (2) 8 (3) 1 (4) 4

Answer: (2)

Solution:

$2^4 \equiv 1(\bmod 15)$

$2^{400} \equiv 1(\bmod 15)$

$2^{403} \equiv 8(\text{mod} 15)$

$\therefore \left\{\frac{2^{403}}{15}\right\} = \frac{8}{15} \Rightarrow K = 15$

10. $\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4} =$

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{2\sqrt{2}\left(1+\sqrt{2}\right)}$ (4) does not exist

Answer: (1)

Solution: Rationalising numerator,

$$\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4} \times \frac{\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}}{\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}}$$

$$= \lim_{y\to 0} \frac{\sqrt{1+y^4}-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)} \times \frac{\sqrt{1+y^4}+1}{\sqrt{1+y^4}+1} = \lim_{y\to 0} \frac{y^4}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

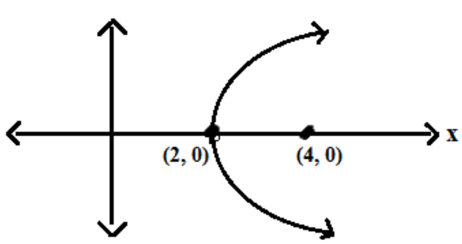
$$= \frac{1}{4\sqrt{2}}$$

11. There is a parabola having axis as $x-$axis, vertex is at a distance of 2 unit from origin & focus is at (4, 0). Which of the following point does not lie on the parabola.

(1) $(6,8)$ (2) $\left(5,2\sqrt{6}\right)$ (3) $\left(8,4\sqrt{3}\right)$ (4) $(4,-4)$

Answer: (1)

Solution:



The equation of parabola is $y^2 = 8(x-2)$

$\therefore$ $(6,8)$ does not lie on this curve

12. Find sum of all possible values of θ in the interval $\left(-\frac{\pi}{2}, \pi\right)$ for which $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely imaginary

(1) $\frac{\pi}{3}$ (2) π (3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{2}$

Answer: (3)

Solution:

$$z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$$

$$= \frac{\left(3-4\sin^2\theta\right)+i\left(8\sin\theta\right)}{1+4\sin^2\theta}$$

For z to be purely imaginary, $\text{Re}(z) = 0$

i.e., $\frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$

$$\Rightarrow \sin^2\theta = \frac{3}{4}$$

As $\theta \in \left(\frac{-\pi}{2}, \pi\right) \Rightarrow \theta = \pm\frac{\pi}{3}, \frac{2\pi}{3}$

$\therefore$ sum of all values of $\theta = \frac{2\pi}{3}$

13. Let $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ Find the value of A^{-50} at $\theta = \frac{\pi}{12}$.

(1) $\begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (3) $\begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$

Answer: (2)

Solution:

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

A is a rotation matrix

$$\therefore\ A^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix} \Rightarrow A^{-50} = \begin{pmatrix} \cos 50\theta & \sin 50\theta \\ -\sin 50\theta & \cos 50\theta \end{pmatrix}$$

$$\therefore\ A^{-50} \text{ at } \theta = \frac{\pi}{12} \text{ is } \begin{pmatrix} \cos\frac{25\pi}{6} & \sin\frac{25\pi}{6} \\ -\sin\frac{25\pi}{6} & \cos\frac{25\pi}{6} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

14. If $(A \oplus B) \wedge (\sim A \Theta B) = A \wedge B$ what should be proper symbol in place of $\oplus$ and Θ to hold the equation

(1) $\wedge$ and $\vee$ (2) $\wedge$ and $\wedge$ (3) $\vee$ and $\vee$ (4) $\vee$ and $\wedge$

Answer: (1)

Solution:

By inspection $\oplus$ represents $\wedge$ and Θ represents $\vee$

A	B	$A \wedge B$	~A	$\sim A \vee B$	$(A \wedge B) \wedge (\sim A \vee B)$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	F	T	T	F
F	F	F	T	T	F

$$\therefore (A \wedge B) \wedge (\sim A \vee B) \equiv A \wedge B$$

15. If $y(x)$ is solution of $x\frac{dy}{dx}+2y=x^2, y(1)=1$ then value of $y\left(\frac{1}{2}\right)=$

(1) $-\frac{49}{16}$ (2) $\frac{49}{16}$ (3) $\frac{45}{8}$ (4) $-\frac{45}{8}$

Answer: (2)

Solution:

$$x.\frac{dy}{dx}+2y=x^2$$

$$\frac{dy}{dx}+\frac{2}{x}y=x$$

This is linear differential equation

I. F $=e^{\int\frac{2}{x}dx}=e^{2\log x}=x^2$

$\therefore$ solution is $x^2y=\int x^3dx$

$$x^2y=\frac{x^4}{4}+c$$

$$y(1)=1 \Rightarrow c=\frac{3}{4}$$

$$\text{at } x=\frac{1}{2} \Rightarrow \frac{1}{4}y=\frac{1}{64}+\frac{3}{4}$$

$$\Rightarrow y=\frac{49}{16}$$

16. From a well shuffled deck of cards, 2 cards are drawn with replacement. If x represent numbers of times ace coming, then value of $P(x=1)+P(x=2)$ is

(1) $\frac{25}{169}$ (2) $\frac{24}{169}$ (3) $\frac{49}{169}$ (4) $\frac{23}{169}$

Answer: (1)

Solution:

$$P(x=1) = {}^2C_1 \times \frac{4}{52} \times \frac{48}{52} = \frac{24}{169}$$

$$P(x=2) = {}^2C_2 \times \left(\frac{4}{52}\right)^2 = \frac{1}{169}$$

$$P(x=1) + P(x=2) = \frac{25}{169}$$

17. If eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is more than 2 when $\theta \in \left(0, \frac{\pi}{2}\right)$ then values of length of latus rectum lies in the interval

(1) $(3, \infty)$ (2) $(1, 3/2)$ (3) $(2, 3)$ (4) $(-3, -2)$

Answer: (1)

Solution:

For hyperbola, $e^2 = 1 + \frac{b^2}{a^2}$

$$= 1 + \tan^2\theta$$

$$= \sec^2\theta$$

$$e > 2 \Rightarrow \sec\theta > 2$$

$$\Rightarrow \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

Length of latus rectum $= \frac{2b^2}{a} = 2\tan\theta\sin\theta$

$$= 2\left(>\sqrt{3}\right)\left(>\frac{\sqrt{3}}{2}\right)$$

>3

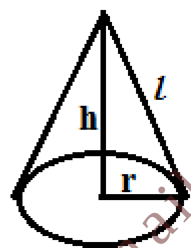
18. If slant height of a right circular cone is 3 cm then the maximum volume of cone is

(1) $2\sqrt{3}\,\pi cm^3$ (2) $4\sqrt{3}\,\pi cm^3$ (3) $\left(2+\sqrt{3}\right)\pi cm^3$ (4) $\left(2-\sqrt{3}\right)\pi cm^3$

Answer: (1)

Solution:

$l=3$



$\Rightarrow r^2+h^2=9$

$\Rightarrow r^2=9-h^2$

$V=\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\pi h\left(9-h^2\right)$

$V=3\pi h-\frac{1}{3}\pi h^3$

For maximum volume, $\frac{dv}{dh}=0$

$\Rightarrow 3\pi-\pi h^2=0$

$\Rightarrow h=\sqrt{3}$

$\Rightarrow r^2=6$

$\therefore$ Volume $=\frac{1}{3}\pi(6)\left(\sqrt{3}\right)=2\sqrt{3}\pi\, cm^3$

19. If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}, x>\frac{3}{4}$ then $x=$

(1) $\frac{\sqrt{145}}{11}$ (2) $\frac{\sqrt{145}}{12}$ (3) $\frac{\sqrt{146}}{10}$ (4) $\frac{\sqrt{146}}{11}$

Answer: (2)

Solution:

$$\cos^{-1}\left(\frac{2}{3x}\right)=\frac{\pi}{2}-\cos^{-1}\left(\frac{3}{4x}\right)$$

$$\cos^{-1}\left(\frac{2}{3x}\right)=\cos^{-1}\left(\frac{\sqrt{16x^2-9}}{4x}\right)$$

$$\frac{2}{3x}=\frac{\sqrt{16x^2-9}}{4x}$$

$$\sqrt{16x^2-9}=\frac{8}{3}\Rightarrow 16x^2-9=\frac{64}{9}$$

$$\Rightarrow x^2=\frac{145}{9\times16}$$

$$\Rightarrow x=\frac{\sqrt{145}}{12}$$

20. If $px+qy+r=0$ represent a family of straight lines such that $3p+2q+4r=0$ then

(1)All lines are parallel (2) All lines are inconsistence

(3) All lines are concurrent at $\left(\frac{3}{4},\frac{1}{2}\right)$ (4) All lines are concurrent at $(3,2)$

Answer: (3)

Solution:

$px+qy+r=0$ … (1)

$3p+2q+4r=0$

$\frac{3p}{4}+\frac{q}{2}+r=0$ … (2)

(1) & (2) are identical

$\frac{x}{\frac{3}{4}}=\frac{y}{\frac{1}{2}}=1$

$(x,y)=\left(\frac{3}{4},\frac{1}{2}\right)$

21. Consider the system of equations $x+y+z=1, 2x+3y+2z=1, 2x+3y+(a^2-1)z=a+1$ then

(1)system has a unique solution for $|a|=\sqrt{3}$ (2) system is inconsistence for $|a|=\sqrt{3}$

(3)system is inconsistence for $a=4$ (4) system is inconsistence for $a=3$

Answer: (2)

Solution:

$x+y+z=1$ … (1)

$2x+3y+2z=1$ … (2)

$2x+3y+(a^2-1)z=a+1$ … (3)

By observation, when $a^2-1=2$

LHS of (2) & (3) are same but RHS different

Hence $a^2=3 \Rightarrow |a|=\sqrt{3}$

$\therefore$ For $|a|=\sqrt{3},$ the system is inconsistent.

22. The value of $3(\cos\theta-\sin\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$ is , where $\theta\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

(1)$13-4\cos^4\theta$ (2)$13-4\cos^6\theta$

(3) $13-4\cos^6\theta+2\sin^4\theta\cos^2\theta$ (4) $13-4\cos^4\theta+2\sin^4\theta\cos^2\theta$

Answer: (2)

Solution:

$$3\left(\cos^2\theta+\sin^2\theta-\sin 2\theta\right)^2+6\left(\sin^2\theta+\cos^2\theta+\sin 2\theta\right)+4\sin^6\theta$$

$$=3\left(1+\sin^2 2\theta-2\sin 2\theta\right)+6+6\sin 2\theta+4\sin^6\theta$$

$$=9+3\sin^2 2\theta+4\sin^6\theta$$

$$=9+3\left(4\sin^2\theta\cos^2\theta\right)+4\left(1-\cos^2\theta\right)^3$$

$$=9+12\cos^2\theta\sin^2\theta+4\left(1-\cos^6\theta-3\cos^2\theta\sin^2\theta\right)$$

$$=9+12\sin^2\theta\cos^2\theta+4-4\cos^6\theta-12\sin^2\theta\cos^2\theta$$

$$=13-4\cos^6\theta$$

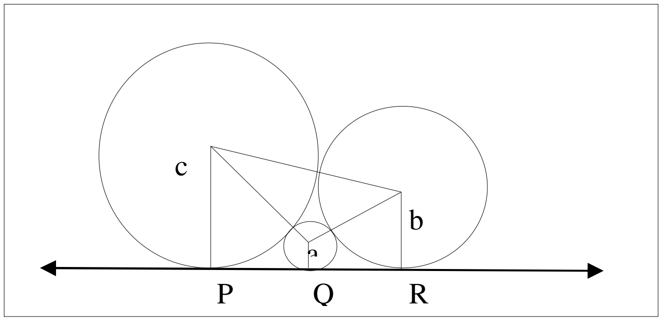
23. 3 circles of radii a, b, c $(a<b<c)$ touch each other externally and have $x-$axis as a common tangent then

(1) a, b, c are in A.P. (2) $\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{c}}$

(3) $\sqrt{a},\sqrt{b},\sqrt{c}$ are in A.P. (4) $\frac{1}{\sqrt{c}}+\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{a}}$

Answer: (4)

Solution:



$$PQ+QR=PR$$

$$\sqrt{(c+a)^2-(c-a)^2}+\sqrt{(b+a)^2-(b-a)^2}=\sqrt{(b+c)^2-(c-b)^2}$$

$$\sqrt{4ac}+\sqrt{4ab}=\sqrt{4bc}$$

Dividing with $\sqrt{4abc}$,

$$\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}=\frac{1}{\sqrt{a}}$$

24. If $f(x)=\frac{1}{x}, f_2(x)=1-x, f_3(x)=\frac{1}{1-x}$ then find $J(x)$ such that $f_2 \, o \, J \, o \, f_1(x)=f_3(x)$

(1) $f_1(x)$ (2) $\frac{1}{x}f_3(x)$ (3) $f_3(x)$ (4) $f_2(x)$

Answer: (3)

Solution:

$$f_2(J(f_1(x)))=f_3(x)$$

$$f_2\left(J\left(\frac{1}{x}\right)\right)=\frac{1}{1-x}$$

$$1-J\left(\frac{1}{x}\right)=\frac{1}{1-x}$$

$$J\left(\frac{1}{x}\right)=1-\frac{1}{1-x}=\frac{x}{x-1}$$

$$J(x)=\frac{1}{1-x}=f_3(x)$$

25. Find the equation of line through $(-4,1,3)$ & parallel to the plane $x+y+z=3$ while the line intersects another line whose equation is $x+y-z=0=x+2y-3z+5$

(1) $\frac{x+4}{-3}=\frac{y-1}{-2}=\frac{z-3}{1}$ (2) $\frac{x+4}{1}=\frac{y-1}{2}=\frac{z-3}{1}$

(3) $\frac{x+4}{-3}=\frac{y-1}{2}=\frac{z-3}{1}$ (4) $\frac{x+4}{-1}=\frac{y-1}{2}=\frac{z-3}{-3}$

Answer: (3)

Solution:

Family of planes containing the line of intersection of planes is $\pi_1+\lambda\pi_2=0$

i.e., $(x+y-z)+\lambda(x+2y-3z+5)=0$

This is passing through $(-4,1,3)$

$\Rightarrow \lambda=-1$

Hence the equation of plane is $y-2z+5=0$

Required line is lie in this plane & is parallel to $x+y+z=5$

$\therefore$ direction of required line $=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 0 & 1 & -2\end{vmatrix}=-3\hat{i}+2\hat{j}+\hat{k}$

$\therefore$ Required line is $\frac{x+4}{-3}=\frac{y-1}{2}=\frac{z-3}{1}$

26. Consider the curves $y=x^2+2$ and $y=10-x^2$. Let θ be the angle between both the curves at point of intersection, then find $|\tan\theta|$

(1) $\frac{8}{15}$ (2) $\frac{5}{17}$ (3) $\frac{3}{17}$ (4) $\frac{8}{17}$

Answer: (1)

Solution:

$x^2+2=10-x^2$

$\Rightarrow x=\pm 2$ & $y=4$

$\therefore$ point of intersection of curves $=(\pm 2,4)$

$y=x^2+2;\ \ y=10-x^2$

$\frac{dy}{dx}=2x;\ \ \frac{dy}{dx}=-2x$

$\frac{dy}{dx}$ at $(\pm\ 2,4)=\pm 4=m_1;\ \frac{dy}{dx}$ at $(\pm 2,4)=\mp 4=m_2$

$\therefore\ |\tan\theta|=\left|\frac{8}{1-16}\right|=\frac{8}{15}$

27. A plane parallel to y-axis passing through line of intersection of planes $x+y+z=1$ & $2x+3y-z-4=0$ which of the point lie on the plane.

(1) $(3,2,1)$ (2) $(-3,0,1)$ (3) $(-3,1,1)$ (4) $(3,1,-1)$

Answer: (4)

Solution:

Required plane is $\pi_1+\lambda\pi_2=0$

$(x+y+z-1)+\lambda(2x+3y-z-4)=0$

$(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-(1+4\lambda)=0$

This is parallel to y-axis $\Rightarrow \lambda=\frac{-1}{3}$

$\therefore$ Required plane is $x+4z+1=0$

By inspection, $(3,1,-1)$ lie in this plane.

28. Find common tangent of the two curves $y^2 = 4x$ and $x^2 + y^2 - 6x = 0$

(1) $y = \frac{x}{3} + 3$ (2) $y = \left(\frac{x}{\sqrt{3}} - \sqrt{3}\right)$ (3) $y = \frac{x}{3} - 3$ (4) $y = \left(\frac{x}{\sqrt{3}} + \sqrt{3}\right)$

Answer: (4)

Solution:

Equation of tangent to the parabola $y^2 = 4x$ is $y = mx + \frac{1}{m}$

$$\Rightarrow m^2x - my + 1 = 0$$

This is also tangent to $x^2 + y^2 - 6x = 0$

i.e., $\left|\frac{3m^2 + 1}{\sqrt{m^4 + m^2}}\right| = 3$

$9m^4 + 1 + 6m^2 = 9m^4 + 9m^2$

$3m^2 = 1 \Rightarrow m = \pm\frac{1}{\sqrt{3}}$

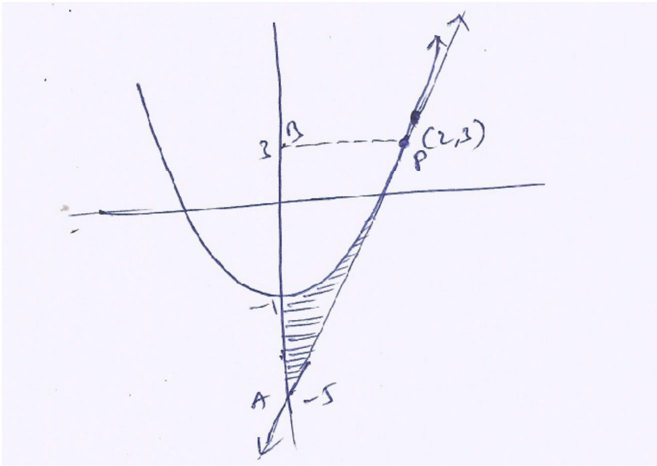
$\therefore$ common tangent is $y = \pm\left(\frac{x}{\sqrt{3}} + \sqrt{3}\right)$

29. If the area bounded by the curve $y = x^2 - 1,$ tangent to it at (2, 3) and y-axis is

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{8}{3}$ (4) 1

Answer: (3)

Solution:



Equation of tangent at (2, 3) is

$$\frac{y+3}{2} = 2x - 1$$

$$y + 3 = 4x - 2$$

$$4x - y = 5$$

Required Area = Ar(Δ PAB) $- \int_{-1}^{3} x_{parabola}\, dy$

$$= \frac{1}{2} \times 2 \times 8 - \int_{-1}^{3} \sqrt{y+1}\, dy$$

$$= 8 - \frac{2}{3}\left((y+1)^{\frac{3}{2}} \right)_{-1}^{3}$$

$$= 8 - \frac{16}{3} = \frac{8}{3}$$